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Mutual inductance bridge for low-temperature thermometry and susceptibility measurements

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We describe a solid-state mutual inductance bridge suitable for low-temperature thermometry and susceptibility measurements. The design features an electronically simulated variable mutual inductance. Particular care is taken to reduce spurious capacitive effects. The bridge is highly stable and linear. It has a resolution of 10^{-4} over the range $15 \mu\text{H}$ to 150 mH , and a noise limited sensitivity of $55 I_p^{-1} \text{ nH}$ with a 1-s time constant, where I_p is the rms primary current in mA. Application to thermometry is discussed.

INTRODUCTION

A useful property for low-temperature thermometry is the susceptibility of a paramagnetic salt that obeys Curie's law at the temperature of interest. Of the several techniques that can be used to measure susceptibility, one of the more popular is an adaptation of the ac Hartshorn bridge¹ in which the paramagnetic salt is placed in the core of a transformer, and the variation of susceptibility with temperature is measured by a change in the mutual inductance. A disadvantage of the Hartshorn bridge is that it requires a precise variable mutual inductance which is both expensive and limited in range. It is possible, however, to overcome this drawback by simulating a variable mutual inductance electronically with an operational amplifier and a fixed inductance. A bridge of this design utilizing vacuum-tube circuitry was reported by Pillinger, Jastram, and Daunt.² This paper describes a solid-state bridge of improved design suitable for low-temperature thermometry and susceptibility measurements.³ The bridge operates at a fixed frequency of 15 Hz and has a resolution of 1 part in 10^4 over the range $15 \mu\text{H}$ to 150 mH . The noise-limited sensitivity of the bridge is $55 I_p^{-1} \text{ nH}$ with a 1-s time constant, where I_p is the rms primary current in the bridge in mA.

I. BRIDGE DESIGN

The basic Hartshorn bridge circuit is shown in Fig. 1. The measuring coil typically is placed in the cryostat and consists of two identical secondaries and a coaxial primary. The secondaries are separated along the axis of the primary coil and are connected in opposition. In principle, the net voltage induced across the two secondaries is zero until a paramagnetic sample is inserted into one of them. The resulting induced emf may then be opposed by the voltage across the secondary of the mutual inductance M and the resistance R . At balance, M is proportional to the real part of the sus-

ceptibility of the sample and R is proportional to the power dissipation. For thermometry it is desirable to minimize the latter which is proportional to ωH^2 , where H is the amplitude of the magnetic field at the sample, and ω is its frequency.

The circuit (Fig 2) that simulates the variable mutual inductance uses an operational amplifier as an adjustable alternating current source to drive a fixed mutual inductance. Negative feedback requires that $i_p f R_p$ equal $i_0 R_m$; thus V_s equals $fM(R_p/R_m)(di_p/dt)$. (C has a negligible susceptance at the bridge frequency and prevents high frequency oscillations in the circuit.) As f varies from zero to one, the effect is to simulate a variable mutual inductance. The linearity and stability of the circuit are determined by the passive components R_m , R_p , and M , provided that the gain and common-mode rejection-ratio of the amplifier are sufficiently high. The resistor R_p used in the actual bridge circuit (Fig. 3) is a $4\frac{1}{2}$ decade non-inductive Kelvin-Varley voltage divider.⁴ R_m is selected by a switch to change the range of the mutual inductance in decade steps. The maximum range of the bridge is set by R_s .

In practice it is hard to construct the measuring coil

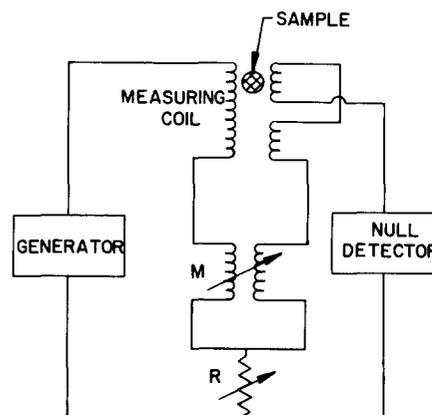


FIG. 1. Basic Hartshorn bridge circuit.

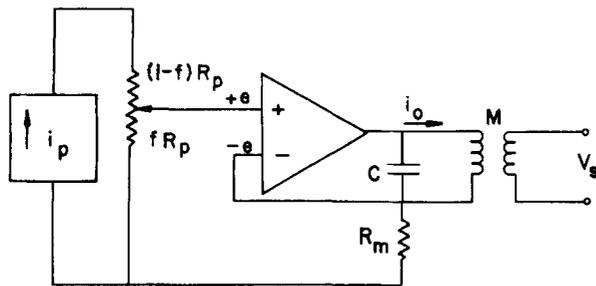


FIG. 2. Circuit which simulates the variable mutual inductance. Negative feedback requires that $e_+ = e_-$, so that $i_p f R_p = i_o R_m$.



FIXED MUTUAL INDUCTANCE M

MEASURING COIL

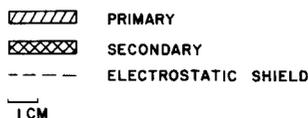


FIG. 4. Cross section of bridge coils. M is wound on a Lucite form, the measuring coil on Bakelite.

with two identical secondaries; furthermore, asymmetries in the cryostat introduce a residual imbalance independent of the sample. For these reasons it is convenient to have two identical mutual-inductance circuits: one to balance the bridge at a fixed temperature (e.g., 4.2 K), and the other to measure the change in the sample susceptibility below that temperature. The fixed mutual inductances are astatically wound⁵ on a rigid, nonmagnetic form. The measuring coil which has been used with this bridge for thermometry is adapted from a design of Abel, Anderson, and Wheatley,⁶ although the bridge is sufficiently versatile to operate with a wide variety of coil designs. Pertinent coil data are summarized in Table I and Fig. 4.

Previous bridge designs^{3,6,7} have exhibited a number of problems that can be traced to capacitive coupling in the bridge, particularly between the primary and secondary circuits. For this reason, electrostatic shields are required between the primaries and secondaries of all coils and transformers in the bridge. A total capacitance between the primary and secondary circuits on the order of 10 pF is acceptable. In addition, capacitance between the secondary circuit and ground can create current paths in the secondary which do not include the null detector. Consequently, even at balance, there may be a current through the measuring coil. The voltage across the measuring coil, therefore, will depend on its resistance, resulting in a shift in the balance when the coil

is cooled. A secondary-to-ground capacitance on the order of 1 nF can be tolerated.

The resistive-balance technique used in the basic Hartshorn bridge, which requires a direct connection between the primary and secondary, exacerbates the capacitive coupling problem. Our bridge uses a technique (Fig. 5) which allows the secondary circuit to float with respect to ground. The voltage across the secondary of T_2 will be in phase with the current in the primary of the bridge if the primary reactance of T_2 is much greater than the closed-loop output impedance of the operational amplifier. Capacitor C_r resonates the transformer and reduces the phase shift to less than 1 mrad. As the phase shift is somewhat dependent on the power level in the transformer, C_r should be selected under actual operating conditions. Since the resistive component of the signal is generally small and not relevant to thermometry, the stability of an air-core transformer is not essential here.

The net voltage across the secondary circuit is ampli-

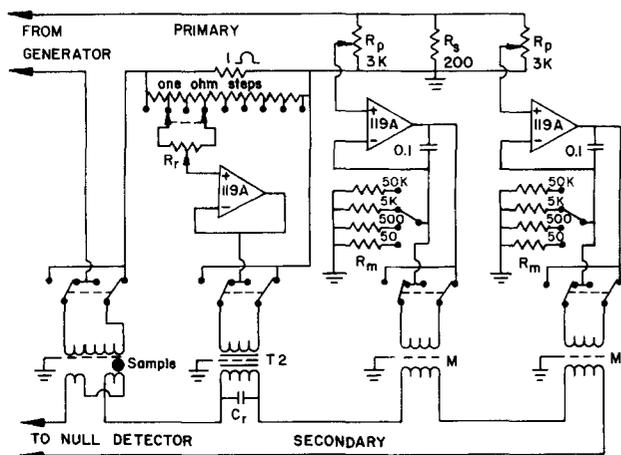


FIG. 3. Bridge circuit. R_r is a 25 Ω ten-turn potentiometer. The operational amplifiers are made by Analog Devices, Norwood, MA.

TABLE I. Bridge coils.

Fixed mutual inductance coils (astatically wound)	
Total mutual inductance	43 mH
Turns—each primary (#28 copper wire)	1000
Turns—each secondary (#28 copper wire)	1000
Capacitance—primary to secondary	14 pF
Capacitance—secondary to shield	260 pF
Measuring coil	
Mutual inductance—primary and one secondary	1 H
Turns—primary (#40 copper wire)	5600
Turns—secondary (#40 copper wire)	8000
Capacitance—primary to secondary	1 pF
Capacitance—secondary to shield	870 pF
Magnetic field in primary	0.5 G/mA
dM/dT (for 1 g of cerous magnesium nitrate at 1 K)	5 μ H/K

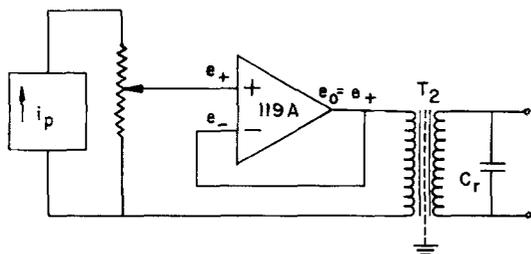


FIG. 5. Resistive balance circuit. T_2 is model A21 made by United Transformer Division, TRW, and is connected with a 3:1 step-down ratio. C_r (16.2 μ F) is chosen to resonate the transformer secondary.

fied and used to determine the balance state of the bridge. The null-detector amplifier (Fig. 6) incorporates a high-quality input transformer T_3 and a low-noise wide-band stage⁸ followed by an active low-pass filter.⁹ A capacitor resonates the secondary of T_3 to improve the signal-to-noise ratio. A useful null detector is a lock-in amplifier; alternatively, the balance condition can be established using a Lissajous figure on an oscilloscope. The scope is driven by a tuned amplifier (twin-T feedback) which in conjunction with the active filter provides an exceedingly high attenuation of line-related noise; nevertheless, 60-Hz pickup is typically many times the stochastic noise in the bridge. Because the 15 Hz operating frequency is phase-locked to the 60 Hz line, all line-related noise is stationary on the Lissajous figure and consequently can be ignored in balancing the bridge. This method has sensitivity comparable to that of a lock-in amplifier with the same time constant. Operation at a subharmonic of the line frequency is also an advantage with a lock-in amplifier, which has extremely high rejection of all even harmonics of the fundamental frequency.¹⁰

The 15-Hz generator (Fig. 7) divides the line frequency to produce a square wave which is integrated and shaped in a tuned amplifier to produce a sine wave. A phase-shift network prior to the frequency divider adjusts the phase of the 60-Hz pickup relative to 15 Hz for the most convenient Lissajous display. A second phase-shift network at 15 Hz drives the horizontal axis of the scope and is adjusted to make the resistive and inductive components of the display orthogonal. The DTL integrated circuits used in the generator are not critical and can be replaced with TTL circuits.

The power supplies for the 15 Hz generator, null-detector amplifier, and bridge operational amplifiers should be regulated independently. T_1 and the power transformers should be physically separated from the rest of the bridge. (Spurious magnetic coupling between bridge elements will not affect the accuracy so long as the coupling is linear in the primary current; however, the leakage flux from T_1 , which operates at a high power level, is nonlinear in i_p .) The fixed mutual inductances, the resistive balance transformer T_2 , and the output transformer T_3 are rigidly mounted using non-magnetic hardware and oriented to minimize magnetic coupling between the components. T_2 and T_3 are mounted in Mu-metal shields, and their cores de-

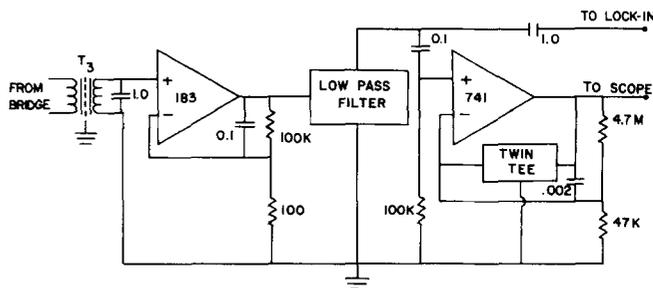


FIG. 6. Null-detector amplifier. T_3 is model 8310 made by James Electronics Inc., Chicago, IL, and is connected with a 1:3 step-up ratio. It is resonated by a 1 μ F capacitor. Amplifier model 183 and the low-pass filter model 9083 are made by Analog Devices, Norwood, MA. The twin-T is model 552-15 made by White Instruments, Austin, TX.

gaussed to reduce microphonic noise. Reversing switches should be placed in the primary circuit. All ground connections are made at one point on the chassis.

II. BRIDGE PERFORMANCE

The stability and linearity of the bridge are determined principally by the quality of the passive components. If R_s , R_p , and R_m are constructed from the same material, their temperature coefficients cancel. In that case the temperature stability of the bridge depends only on the mutual inductances. Cooling the measuring coil from 300 to 4.2 K shifts the resistive balance by 200% and the inductive balance by 0.7%. These shifts are probably due to differential contraction and self-capacitance in the measuring coil. At 4.2 K the inductive balance is unchanged by a 1 k Ω increase in the resistance of the secondary circuit, while the resistive balance shifts by 2%.

As the Kelvin-Varley voltage divider and the hand-selected range resistors are accurate to one part in 10^4 , it is likely that the bridge is linear to the same degree. The linearity can be checked to one part in 10^3 by using one mutual-inductance circuit to set an imbalance which is then balanced by the other circuit on each of its decade ranges. The inductive balance of the bridge is orthogonal to the resistive balance as long as the null

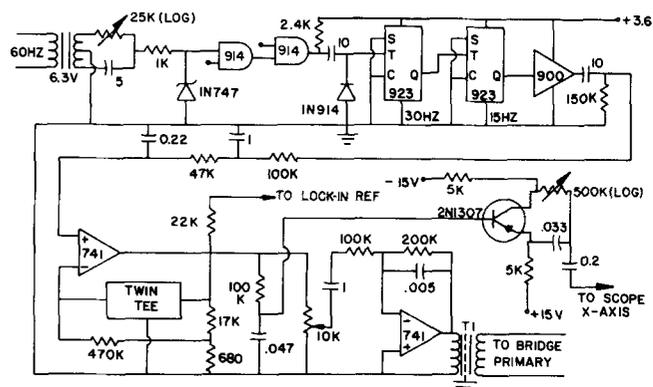


FIG. 7. The 15 Hz generator. T_1 is model LS56 made by United Transformer Division, TRW, and is connected with a 3:1 step-down ratio. The 900 series are Fairchild DTL circuits. The twin-T is Model 552-15 made by White Instruments, Austin, TX.

detector is not overloaded. Finally, no dependence of the bridge balance on primary current is observed.

The sensitivity of the bridge depends on the resistance of the secondary circuit. With the secondary of the measuring coil shorted, the sensitivity is limited by amplifier noise and is $55 I_p^{-1}$ nH with a 1-s time constant, where I_p is the rms primary current in mA. With the measuring coil included in the secondary circuit, the sensitivity at low temperatures is approximately the shorted-coil value. In our particular application, however, transient eddy currents induced by small mechanical vibrations of the measuring coil approximately doubled the noise at 15 Hz.

Using a paramagnetic salt for thermometry which obeys Curie's law ($\chi \propto T^{-1}$), the fractional temperature resolution $\Delta T/T$ is equal to $\Delta M/M$, where M is the total mutual inductance due to the presence of the salt, and ΔM is the minimum detectable mutual inductance. In practice, ΔM is determined by the noise-limited sensitivity of the bridge and is inversely proportional to the primary current or the magnetic field at the salt. Experimental considerations such as saturation or heating of the salt place a practical upper limit on the magnetic field. However, these effects are usually important only at the lowest temperatures, where M is large. It is possible to calibrate the salt above 1 K using a high magnetic field, and maintain the same fractional temperature resolution below 1 K while operating the bridge at reduced current. For example, one gram of cerous magnesium nitrate ($\chi = 4.14 \times 10^{-4} T^{-1}$ emu/g) produces a 5 μ H change in the mutual inductance of the measuring coil at 1 K. This salt can therefore be calibrated between 1 and 4 K using a 20-G field to a precision of 0.1%. The same resolution can be maintained at 10 mK with a 0.05-G field.

ACKNOWLEDGMENTS

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- ¹ G. K. White, *Experimental Techniques in Low Temperature Physics* (Oxford U.P., Oxford, 1968), 2nd ed., p. 270.
- ² W. L. Pillinger, P. S. Jastram, and J. G. Daunt, *Rev. Sci. Instrum.* **29**, 159 (1958). A solid-state adaptation of their bridge has been reported by J. M. Dundon and M. E. Kretschmar, *Rev. Sci. Instrum.* **49**, 406 (1978).
- ³ S. C. Whitmore, T. M. Sanders, Jr., and S. R. Ryan, *Proceedings of the 11th International Conference on Low Temperature Physics*, edited by J. F. Allen, D. M. Findlayson, and D. M. McCall (University of St. Andrews, St. Andrews, 1968), p. 661.
- ⁴ Model X26A, Electro Scientific Industries, Portland, OR. A ten-turn potentiometer proved unsatisfactory in this application.
- ⁵ Astatic coils are wound with two coaxial sets of primaries and secondaries, each set contributing half of the required mutual inductance. The secondaries are connected to cancel any voltage induced by a uniform stray field common to both of them, while the primaries are connected to give a net mutual inductance.
- ⁶ W. R. Abel, A. C. Anderson, and J. C. Wheatley, *Rev. Sci. Instrum.* **35**, 444 (1964).
- ⁷ This problem has also been discussed by A. C. Anderson, R. E. Peterson, and J. E. Robichaux, *Rev. Sci. Instrum.* **41**, 528 (1970), and by L. J. M. van de Klundert, C. de Rooij, M. Caspari, and L. C. van der Marel, *Cryogenics* **15**, 577 (1975).
- ⁸ Model 183, Analog Devices, Inc., Norwood MA. Model 183 is currently superseded by model 184. The turns ratio of T_3 is selected to optimize the signal-to-noise ratio for model 183 with a source impedance of 1.5 k Ω , the impedance of the bridge secondary when the measuring coil is at 4.2 K. The circuit has a gain of 3×10^3 and an equivalent input-noise voltage of 10 nV Hz^{-1/2}.
- ⁹ Model 9083, Analog Devices, Inc., Norwood, MA. This is a tunable low-pass filter with a four-pole Tchebyscheff response which provides -32 dB rejection at 30 Hz (1800 rpm electric motor interference), -46 dB at 45 Hz (third harmonic distortion from transformer T_1), and -57 dB at 60 Hz.
- ¹⁰ N. T. Larsen, *Rev. Sci. Instrum.* **33**, 1200 (1962).

